Nonfactorizable $B \to \chi_{c0} K$ decay and QCD factorization

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We study the unexpectedly large rate for the factorization-forbidden decay $B \to \chi_{c0} K$ within the QCD factorization approach. We use a non-zero gluon mass to regularize the infrared divergences in vertex corrections. The end-point singularities arising from spectator corrections are regularized and carefully estimated by the off-shellness of quarks. We find that the contributions arising from the vertex and leading-twist spectator corrections are numerically small, and the twist-3 spectator contribution with chiral enhancement and linear end-point singularity becomes dominant. With reasonable choices for the parameters, the branching ratio for $B \to \chi_{c0} K$ decay is estimated to be in the range $(2-4) \times 10^{-4}$, which is compatible with the Belle and BaBar data.

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B meson exclusive decays to hadrons with charmonium are interesting in studies of both strong interaction dynamics and CP violation. The naively factorizable decays [1] such as $B \to J/\psi K$ [2, 3], $B \to \eta_c K$ [4], and $B \to \chi_{c1} K$ [5] were studied in the QCD factorization approach [6] in which the nonfactorizable vertex and spectator corrections were also estimated.

To further explore the nonfactorizable contributions it is worth studying the factorization-forbidden decays such as $B \to \chi_{c0} K$. Recently, $B \to \chi_{c0} K$ decay has been observed by Belle [7, 8] and BaBar [9] with surprisingly large branching ratio which is even comparable to that of the factorization-allowed decay $B \to \chi_{c1} K$:

$$\begin{aligned}
\mathrm{Br}(B^+ \to \chi_{c0} K^+) &= (6.0^{+2.1}_{-1.8} \pm 1.1) \times 10^{-4} \text{ [7]}, \\
&= (1.96 \pm 0.35 \pm 0.33) \times 10^{-4} \text{ [8]}, \\
\mathrm{Br}(B^\pm \to \chi_{c0} K^\pm) &= (2.7 \pm 0.7) \times 10^{-4} \text{ [9]}. \qquad (1)
\end{aligned}$$

To explain the large decay rate of $B \to \chi_{c0} K$, the final state re-scattering mechanism was suggested [10]. On the other hand, with the Light-Cone Sum Rules [11] the nonfactorizable contributions were found to be too small to accommodate the observed $B \to \chi_{c0} K$ decay rate.

In fact, in Ref. [5, 12] within the QCD factorization approach it was found that for the $B \to \chi_{c0} K$ decay, there exist both the infrared (IR) divergences in the vertex corrections and the end-point singularities in the leading twist spectator corrections. This implies that large nonfactorizable contributions may come from soft gluon exchanges. As argued in [12], unlike the inclusive B decays to charmonium, where the IR divergences can be factorized into the color-octet matrix elements associated with the higher Fock states of color-octet $c\bar{c}$ with soft gluons [13] in nonrelativistic QCD (NRQCD) [14], the IR divergences in the exclusive two-body decays are difficult to be factorized.

At the qualitative level, the results of Ref. [13] may suggest that some fraction of the large color-octet contribution in the inclusive B decays to charmonium does in fact end up in two-body decay modes. That is, the soft gluon emitted by the color-octet $c\bar{c}$ pair may be reabsorbed by the quarks in the kaon or B meson, leading to a possible connection between the color-octet contribution and the infrared behavior of vertex and spectator corrections in QCD factorization approach. So qualitative estimates of these soft gluon contributions are important for understanding the large branching ratios of both $B \to \chi_{c0} K$ and $B \to J/\Psi K$. Furthermore, since the s quark emitted from the weak vertex moves fast in the B meson rest frame, we may expect that the soft gluon exchange is dominated by that between the $c\bar{c}$ pair and the spectator quark.

In order to estimate the soft gluon contributions in these exclusive decays, we may let the charm quark be off the mass shell or give the gluon a mass, and then use the binding energy or gluon mass to regulate the IR behavior. Introducing the binding energy [15] or momentum cutoff [16] was an useful way in estimating the inclusive annihilation rates of h_c and χ_{c1} before the NRQCD factorization theory is developed. Even after that, this approach is still widely used in quarkonium phenomenology for qualitative estimates (see, e.g., [17]), though it is not a rigorous theory. Recently, the binding energy regularization was suggested in [18]. In the present paper, we will estimate the non-factorizable decay rate for $B \to \chi_{c0} K$ by using the gluon mass regularization for the IR divergences, which is equivalent to the binding energy regularization in physical nature. The main differences between our calculations and those in Ref. [18] are the treatments of the end-point singularities in spectator interactions, which play the most important role in numerical evaluations.

We treat charmonium as a color-singlet nonrelativistic (NR) $c\bar{c}$ bound state. Let p be the total momentum of the charmonium and $2q$ be the relative momentum between c

and $\bar{c}$ quarks, then $v^2 \sim 4q^2/p^2 \sim 0.25$ can be treated as a small expansion parameter [14]. For P-wave charmonium χ_{c0}, because the wave function at the origin $\mathcal{R}_1(0) = 0$, which corresponds to the zeroth order in q, we must expand the amplitude to first order in q. Thus we have (see, e.g., [19])

$$\mathcal{M}(B \to \chi_{c0} K) = \sum_{L_z, S_z} \langle 1L_z; 1S_z|00\rangle \int \frac{\mathrm{d}^4 q}{(2\pi)^3} q_\alpha \delta(q^0) \times \psi^*_{1M}(q) \mathrm{Tr}[\mathcal{O}^\alpha(0) P_{1S_z}(p,0) + \mathcal{O}(0) P^\alpha_{1S_z}(p,0)], \quad (2)$$

where $\mathcal{O}(q)$ represents the rest of the decay matrix element and can be further factorized as the product of $B \to K$ form factors and a hard kernel or as the convolution of a hard kernel with light-cone wave functions of B and K mesons, within the framework of QCD factorization approach. The spin-triplet projection operator $P_{1S_z}(p,q)$ is constructed in terms of quark and antiquark spinors as[1]

$$\begin{aligned} P_{1S_z}(p,q) &= \sqrt{\frac{3}{m_c}} \sum_{s_1,s_2} v(\frac{p}{2} - q, s_2) \bar{u}(\frac{p}{2} + q, s_1) \langle s_1; s_2|1S_z\rangle \\ &= -\sqrt{\frac{3}{4M^3}} (\frac{\not p}{2} - \not q - \frac{M}{2}) \not\epsilon^*(S_z)(\not p + M) \\ &\quad \times (\frac{\not p}{2} + \not q + \frac{M}{2}), \end{aligned} \quad (3)$$

and

$$\mathcal{O}^\alpha(0) = \frac{\partial \mathcal{O}(q)}{\partial q_\alpha}|_{q=0}, \quad (4)$$

$$P^\alpha_{1S_z}(p,0) = \frac{\partial P_{1S_z}(p,q)}{\partial q_\alpha}|_{q=0}. \quad (5)$$

In Eq. (3) we take charmonium mass $M \simeq 2m_c$ in NR limit. Here m_c is the charm quark mass.

The integral in Eq. (2) is proportional to the derivative of the P-wave wave function at the origin

$$\int \frac{\mathrm{d}^3 q}{(2\pi)^3} q^\alpha \psi^*_{1M}(q) = i\varepsilon^{*\alpha}(L_z) \sqrt{\frac{3}{4\pi}} \mathcal{R}'_1(0), \quad (6)$$

and we will use the following polarization relation for χ_{c0}:

$$\sum_{L_Z S_Z} \varepsilon^{*\alpha}(L_z) \epsilon^{*\beta}(S_z) \langle 1L_z; 1S_z|00\rangle = \frac{1}{\sqrt{3}} (-g^{\alpha\beta} + \frac{p^\alpha p^\beta}{M^2}). \quad (7)$$

Being contrary to χ_{c0}, the K meson is described relativistically by the light-cone distribution amplitudes (LCDAs) [6]:

$$\begin{aligned} &\langle K(p')|\bar{s}_\beta(z_2)\, d_\alpha(z_1)|0\rangle = \\ &\frac{if_K}{4} \int_0^1 dx e^{i(y\, p'\cdot z_2 + \bar{y}\, p'\cdot z_1)} \Big\{ \not p'\, \gamma_5\, \phi_K(y) \\ &-\mu_K \gamma_5 (\phi^p_K(y) - \sigma_{\mu\nu}\, p'^\mu (z_2 - z_1)^\nu \, \frac{\phi^\sigma_K(y)}{6}) \Big\}_{\alpha\beta}, \end{aligned} \quad (8)$$

where y and $\bar{y} = 1 - y$ are the momentum fractions of the s and $\bar{d}$ quarks inside the K meson respectively, and the chirally enhanced mass scale $\mu_K = {m_K}^2/(m_s(\mu) + m_d(\mu))$ is comparable to m_b, which ensures that the twist-3 spectator interactions are numerically large, though they are suppressed by $1/m_b$ (see [20]). The twist-2 LCDA $\phi_K(y)$ and the twist-3 LCDA $\phi^p_K(y)$ and $\phi^\sigma_K(y)$ are symmetric under $y \leftrightarrow \bar{y}$ in the $SU(3)$ symmetry limit. In practice, we choose the asymptotic forms for these LCDAs,

$$\phi_K(y) = \phi^\sigma_K(y) = 6y(1-y), \qquad \phi^p_K(y) = 1, \quad (9)$$

The effective Hamiltonian relevant for $B \to \chi_{c0} K$ is [21]

$$\mathcal{H}_{\mathrm{eff}} = \frac{G_F}{\sqrt{2}} \Big(V_{cb} V^*_{cs} (C_1 \mathcal{O}_1 + C_2 \mathcal{O}_2) - V_{tb} V^*_{ts} \sum_{i=3}^{6} C_i \mathcal{O}_i \Big), \quad (10)$$

where G_F is the Fermi constant, C_i are the Wilson coefficients, and $V_{q_1 q_2}$ are the CKM matrix elements. We do not include the effects of the electroweak penguin operators since they are numerically small. Here the relevant operators $\mathcal{O}_i$ are given by

$$\begin{aligned} \mathcal{O}_1 &= (\bar{s}_\alpha b_\beta)_{V-A} \cdot (\bar{c}_\beta c_\alpha)_{V-A}, \\ \mathcal{O}_2 &= (\bar{s}_\alpha b_\alpha)_{V-A} \cdot (\bar{c}_\beta c_\beta)_{V-A}, \\ \mathcal{O}_{3,5} &= (\bar{s}_\alpha b_\alpha)_{V-A} \cdot \textstyle\sum_q (\bar{q}_\beta q_\beta)_{V\mp A}, \\ \mathcal{O}_{4,6} &= (\bar{s}_\alpha b_\beta)_{V-A} \cdot \textstyle\sum_q (\bar{q}_\beta q_\alpha)_{V\mp A}, \end{aligned} \quad (11)$$

where α, β are color indices and the sum over q runs over u, d, s, c and b. Here $(\bar{q}_1 q_2)_{V\pm A} = \bar{q}_1 \gamma_\mu (1 \pm \gamma_5) q_2$.

According to [6] all non-factorizable corrections are due to the diagrams in Fig.1. These corrections, with operators $\mathcal{O}_i$ inserted, contribute to the amplitude $\mathcal{O}(q)$ in Eq. (2), where the external lines of charm and anti-charm quarks have been truncated. Taking non-factorizable corrections in Fig.1 into account, the decay amplitude for $B \to \chi_{c0} K$ in QCD factorization is

$$i\mathcal{M} = \frac{G_F}{\sqrt{2}} \Big[V_{cb} V^*_{cs} C_1 - V_{tb} V^*_{ts} (C_4 + C_6) \Big] \times A, \quad (12)$$

and the coefficient A is given by

$$A = \frac{i6\mathcal{R}'_1(0)}{\sqrt{3\pi M}} \frac{\alpha_s}{4\pi} \frac{C_F}{N_c} (1-z) F_1(M^2) \frac{m_B^2}{M} \cdot \big(fI + fII \big). \quad (13)$$

Here N_c is the number of colors, $C_F = (N_c^2 - 1)/(2N_c)$, and $F_{0,1}$ are the $B \to K$ form factors. We have used

[1] To construct the spin-singlet projection operator $P_{00}(p,q)$, one only needs to replace $\not\epsilon^*$ by γ_5 in Eq.(3).

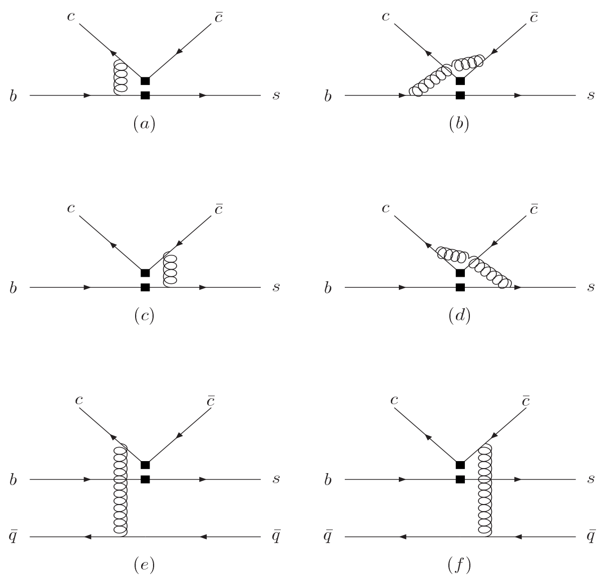

FIG. 1: Feynman diagrams for vertex and spectator corrections to $B \to \chi_{c0} K$.

the relation $F_0(M^2)/F_1(M^2) = 1 - z$ [2, 3], where $z = M^2/m_B^2 \approx 4m_c^2/m_b^2$, to simplify the amplitude in (13). The function fI is calculated from the four vertex diagrams (a, b, c, d) in Fig.1, and fII is calculated from the two spectator diagrams (e, f) in Fig.1. The function fII receives contributions from both twist-2 and twist-3 LCDAs of the K-meson, and we can simply symbolize them as fII^2 and fII^3, respectively.

In the statement of color transparency [6], the IR divergences should be cancelled between diagrams (a,b), (c,d) and (e,f) respectively in Fig.1. But it is not true when the emitted meson is a P-wave charmonium, say, χ_{c0}. The soft gluons, which are emitted from a certain source, couple to the charm and anti-charm quarks through both color charge and color dipole interactions. For the color-singlet charm quark pair, the total charge is zero, but the color dipole interactions, which are proportional to the relative momentum q, give the leading order contributions in both $1/m_b$ power expansion and NR expansion (see Eq. (2)). As a result, there exist IR divergences in fI, while $fII^{2,3}$ suffer from logarithmic and linear endpoint singularities if the asymptotic form of kaon LCDAs are used.

Now we can express the function fI as the Feynman parameter integrals

$$fI = -\int_0^1 dx \int_0^{1-x} dy \ (d + c), \tag{14}$$

where the fuctions d and c are written by

$$\begin{aligned} d \ &= \ \frac{2(1-z)}{(x+\frac{y}{2})(x+\frac{zy}{2})} - \frac{(1-z)(1+z)xy}{((x+\frac{y}{2})(x+\frac{zy}{2}))^2} \\ &+ \frac{2(1-z)(1-x)}{\frac{y}{2}((z-1)x+\frac{zy}{2})} + \frac{(1-z)^2 xy(1-x)}{(\frac{y}{2}((z-1)x+\frac{zy}{2}))^2} \ , \qquad (15) \\ c \ &= \ \frac{-y(z(3y+4)+2)+x(3y-z(9y+4)-2)}{(x+\frac{y}{2})(x+\frac{zy}{2})} \\ &+ \frac{\frac{1-z}{2}xy((2z-6)x^2+(6-zy-3y)x+y(z+2-zy))}{((x+\frac{y}{2})(x+\frac{zy}{2}))^2} \\ &+ \frac{y(3x-2+z(3y-3x+2))}{\frac{y}{2}((z-1)x+\frac{zy}{2})} \\ &+ \frac{\frac{z-1}{2}xy^2(-3x+2+z(3x+y-3))}{(\frac{y}{2}((z-1)x+\frac{zy}{2}))^2} \ . \end{aligned}$$

From Eq. (15), we can see that the IR poles are all included in the function d before we integrate it completely. The first two and the last two terms in d come from diagrams (a,b) and (c,d) in Fig.1, respectively. The divergent integrals in Eq. (14) can be regularized by non-zero gluon mass m_g, as we have mentioned above. And the gluon mass pole in the divergent integral is given by

$$\begin{aligned} &\int_0^1 dx \int_0^{1-x} dy \ d \ = \\ &-\frac{8z(1-z+\ln z)}{(1-z)^2} \ln\left(\frac{m_g^2}{m_b^2}\right) + \text{finite terms}. \end{aligned} \tag{16}$$

Note that the infrared divergence would vanish if $z \to 0$ (i.e., if treating the charm quark as a light quark).

To derive functions $fII^{2,3}$, we extract the light-cone projector of K meson in momentum space from Eq. (8),

$$\begin{aligned} M^K_{\alpha\beta}(p') \ &= \ \frac{if_K}{4}\Big\{ \not p' \gamma_5 \, \phi_K(y) \\ &-\mu_K \gamma_5 \big(\phi_K^p(y) - i\sigma_{\mu\nu} \, p'^{\mu} \frac{\partial}{\partial k_{2\nu}} \frac{\phi_K^\sigma(y)}{6}\big)\Big\}_{\alpha\beta}, \end{aligned} \tag{17}$$

where $k_{2(1)}$ is the momentum of the anti-quark (quark) in K meson, and the derivative acts on the hard-scattering amplitudes in momentum space.

Using the projector given in (17) and eliminating $\phi_K^{p,\sigma}(y)$ by (9), we get the explicit form of fII^2 and fII^3:

$$fII^2 = a\int_0^1 d\xi \frac{\phi_B(\xi)}{\xi} \int_0^1 dy \frac{\phi_K(y)}{\bar{y}^2}[-2z + (1-z)\bar{y}], \tag{18}$$

$$fII^3 = \frac{a \cdot r_K}{1-z} \int_0^1 d\xi \frac{\phi_B(\xi)}{\xi} \int_0^1 dy \frac{1}{\bar{y}^2}[3z - (1-z)\bar{y}], \tag{19}$$

where the factor a is defined as

$$a = \frac{8\pi^2 f_K f_B}{N_c(1-z)^2 m_B^2 F_1(M^2)} \ . \tag{20}$$

Here, due to our definition of fII in (13), the form factor F_1 is present in the denominator.

In Eqs. (18, 19), ξ is the momentum fraction of the spectator quark in the B meson and

$$r_K(\mu) = 2m_K^2/[m_b(\mu)(m_s(\mu)+m_d(\mu))] \qquad (21)$$

is of order one and can not be neglected. The integral over ξ is conventionally parameterized as [6]

$$\int_0^1 d\xi \frac{\phi_B(\xi)}{\xi} = \frac{m_B}{\lambda_B} \, . \qquad (22)$$

If we choose the asymptotic form of the LCDAs of kaon in Eq. (9), we will find logarithmic (linear) singularities in fII^2 (fII^3). As we have mentioned before, these singularities came from the color dipole interactions between the soft gluons and the P-wave charm quark pair, just like what happened in the vertex corrections.

Our result of fII^2 is consistent with the previous ones [12, 18]. However, our function fII^3 is different from that in Ref. [18] because they used a twist-3 light-cone projector

$$M^K_{\alpha\beta} = \frac{if_K}{4}\Big\{\not p'\,\gamma_5\,\phi_K(y) - \mu_K\gamma_5 \frac{\not k_2 \not k_1}{k_2\cdot k_1}\phi^p_K(y)\Big\}_{\alpha\beta}, \qquad (23)$$

which could be derived from Eq. (17) by adopting an integration by parts on y and dropping the boundary terms. If one simply parameterizes the linear singularities in Eq. (19) as

$$\int \frac{dy}{y^2} = \frac{m_B}{\Lambda_h} \, , \qquad (24)$$

where $\Lambda_h \sim 500\,\mathrm{MeV}$ is the infrared cutoff, we will find a large difference between our results and those in Ref. [18]. Since the decay rate is very sensitive to the numerical value of fII^3 as one can see in the following, we should be very careful in regularizing these singularities.

The key point here is that the boundary terms have the form $\frac{\phi^\sigma_K(y)}{y^2}|_0^1$ and can not be dropped safely, when the asymptotic form of $\phi^\sigma_K(y) = 6y\bar y$ is inserted. So the integration by parts is not well-defined when one simply parameterizes the linear singularities as in Eq. (24) and in Ref. [18]. That is why our result of fII^3 is different from that in Ref. [18].

The above problem comes from the observation that the product of virtualities of the propagators in Fig.1 (e,f) goes to zero faster than $\phi^\sigma_K(y)$ in the end point regions. However, if we regularize all these small-virtualities carefully (i.e., introducing small off-shellness or transverse momenta for quarks and gluons), as suggested in Ref. [22], we can see that the boundary terms are exactly zero,

$$\frac{y^n\phi^\sigma_K(y)}{(y+\lambda)^{n+2}}|_0^1 = 0, \qquad n = 0, 1, 2\, \ldots \, , \qquad (25)$$

and then the integration by parts is well-defined. That is, the two projectors are equivalent if and only if one regularizes the linear singularities properly, e.g.,

$$\int \frac{dy}{y^2} \to \int \frac{dy}{(y+\lambda)^2} = \frac{1}{\lambda} - 1 + O(\lambda) \, ,$$
$$\int \frac{ydy}{y^3} \to \int \frac{ydy}{(y+\lambda)^3} = \frac{1}{2\lambda} - 1 + O(\lambda)\, \ldots \, , \qquad (26)$$

where the relative off-shellness λ should be of order of Λ_h/m_B. In practice, one needs to be careful and note that the integral kernels $1/y^2$ and y/y^3 give different contributions in the scheme described by Eq. (26), although superficially they are the same in the expression of Eq. (19) and in the parametrization of Eq. (24).

It is worth emphasizing that this scheme is more physical than the one in Eq. (24) since the off-shellness or transverse momenta of quarks and gluons are naturally serve as infrared cutoffs when $y \to 0$. Furthermore, it is a proper scheme to realize the factorization for the electromagnetic form factors of π at twist-3 level [22].

To determine the value of λ, we introduce the binding energy $b \simeq M - 2m_c > 0$ [18] for χ_{c0}, and then the virtuality of the quark line in Fig.1(e) or (f) goes to

$$(p_c + \bar y p')^2 - m_c^2 \simeq \frac{(1-z)}{2} m_B^2 (\bar y + \frac{z}{1-z}\frac{b}{M}). \qquad (27)$$

Here it is evident that the last term in the last parentheses plays the role of λ. In the APPENDIX we will show that the relative off-shellness determined by the gluon propagator is of the same sign and the same order as λ. In practice, we use

$$\lambda = \frac{z}{1-z}\frac{b}{M}\, . \qquad (28)$$

Following the scheme in Eq. (26) and using the asymptotic forms of LCDAs in Eq. (9), we reexpress the function fII^3 as

$$fII^3 = \frac{a\cdot r_K}{1-z}\cdot\frac{m_B}{\Lambda_B}\int_0^1 dy\Big[\frac{3z}{(y+\lambda)^2} - \frac{3zy + (1-4z)y^2}{(y+\lambda)^3} + \frac{3zy^2 - 3zy^3}{(y+\lambda)^4}\Big]. \qquad (29)$$

For simplicity, here we have used the same relative off-shellness λ to regularize each factor y in denominators. If we set $\lambda = 0$ in Eq. (29), this expression will fall back upon the form given in Eq. (19). We also derive the function by using the projector in Eq. (23), and get a different expression,

$$fII^3 = \frac{a\cdot r_K}{1-z}\cdot\frac{m_B}{\Lambda_B}\int_0^1 dy\Big[\frac{3z}{(y+\lambda)^2} - \frac{zy + (1-z)y^2}{(y+\lambda)^3}\Big]. \qquad (30)$$

Similarly, up to a universal normalization factor, the function fII^3 will fall back on the same form given in

Ref. [18] if λ is set to be zero. Completing the integrals in Eq. (29) and Eq. (30), we get the same result

$$fII^3 = \frac{a \cdot r_K}{1-z}\frac{m_B}{\Lambda_B}[\frac{5z}{2\lambda}+(1-z)ln\lambda+\frac{1}{2}(3-7z)+O(\lambda)], \quad (31)$$

as it should be. Again, we should emphasize that the emergence of infrared divergences and end-point singularities in decay amplitudes partly destroys the factorization assumption, and the soft interaction mechanism may be dominant in this decay mode. Before the divergences are removed or absorbed in some other factorization schemes, Eq. (31) can reasonably serve as a model dependent estimation for soft gluon effects contributing to twist-3 spectator interactions. Furthermore, because of the linear singularities in Eq. (31), fII^3 is not power suppressed in $1/m_B$, rather it is chirally and kinematically enhanced.

To be consistent with fII^3, we regularize fII^2 in the same scheme and get

$$fII^2 = a \cdot \frac{m_B}{\Lambda_B}[12zln\lambda + 21z + O(\lambda)]. \quad (32)$$

For numerical estimates, we use the following input parameters with two values for the QCD scale $\mu = \sqrt{m_b\Lambda_h} \approx 1.45$ Gev and $\mu = m_b \approx 4.4$ Gev:

$$\begin{aligned}
&M = 3.42\text{ GeV}, m_B = 5.28\text{ GeV}, \lambda_B = 300\,\text{MeV}, \\
&f_B = 216\text{ MeV[23]}, f_K = 160\text{ MeV}, F_1(M^2) = 0.75\text{ [24]}, \\
&\mathcal{R}_1^{'}(0) = \sqrt{0.075}\,\text{GeV}^{5/2}\text{[25]},\ C_1(\mu) = 1.239(1.114), \\
&C_4(\mu) = -0.046(-0.027),\ C_6(\mu) = -0.068(-0.033), \\
&\alpha_s(\mu) = 0.34(0.22),\ r_K(\mu) = 0.85(1.3),\ \lambda = 0.087. \quad (33)
\end{aligned}$$

In Eq. (33) the μ-dependent quantities at $\mu = 1.45$ Gev ($\mu = 4.4$ Gev) are shown without (with) parentheses. The Wilson coefficients C_i are evaluated at leading order by renormalization group analysis [21], since the amplitudes in Eq. (13) are of leading order in α_s.

The numerical results of fI are listed in Tab.I with the gluon mass varying from 200 Mev to 500 Mev (the typical scale of Λ_{QCD}). Comparing fI with the values of $fII^{2,3}$, we find that for $B \to \chi_{c0}K$, both fI and fII^2 are small and they are partially canceled. As a consequence, the prediction for Br($B \to \chi_{c0}K$) would be about an order of magnitude smaller than Eq. (1) if only the leading-twist functions were used (see Tab.II). However, the chirally enhanced twist-3 contribution is numerically large and makes the predicted decay rate to be comparable to the experimental data Eq. (1).

Our predictions for the branching ratio of $B \to \chi_{c0}K$ are listed in Tab.II, and the values in the parentheses are the results evaluated by using leading-twist contributions only. From Tab.II we can see that the results are not very sensitive to the value of the gluon mass. For comparison, we follow the available results in Ref. [18], where the IR divergences are regularized by binding energy, to evaluate fI and give Br($B \to \chi_{c0}K$)=22(16)$\times 10^{-5}$ for $\mu = 1.45(4.4)$ Gev. Again, the conclusion is similar to the gluon mass scheme. We see that by treating the singularities in spectator interactions properly, we can obtain the large experimental rate of $B \to \chi_{c0}K$ without introducing any unknown imaginary part for spectator interactions [18].

In summary, We have studied the factorization-forbidden decay $B \to \chi_{c0}K$ within the framework of the QCD factorization approach. We use the gluon mass to regularize the infrared divergence in vertex corrections. The end-point singularities arising from spectator interactions are regularized and estimated carefully by the off-shellness of quarks in the small virtuality regions. We find that for this decay the contributions from the vertex and leading-twist spectator corrections are numerically small, and the twist-3 spectator contribution with chiral enhancement and linear end-point singularity becomes dominant. With reasonable choices for the parameters, the $B \to \chi_{c0}K$ decay branching ratio is estimated to be in the range of $(2-4)\times 10^{-4}$, which is compatible with the Belle and BaBar data. We would like to point out that there are other B exclusive decays to charmonia such as $B \to \chi_{c2}K$, $B \to h_cK$ as well as $B \to \psi(3770)K$, and it is worth studying those decays to see if the soft spectator interactions are also the dominant mechanisms[26].

Note. Since this result was reported in arXiv:hep-ph/0502240, the $B \to \chi_{c0}K$ decay has also been studied with k_T-factorization in the PQCD approach [27], in which the vertex corrections are ignored and the spectator corrections are found to give a large enough decay rate in comparison with experiment. Their result is consistent with ours in the sense that in both approaches the vertex corrections are found to be small, and the spectator corrections give dominant contributions to the decay rate. For the vertex corrections, the authors of Ref. [27] take them into account through the variation of renormalization scale for the factorizable contributions, while we use the gluon mass (or the binding energy) to regularize the IR divergence

TABLE I: Functions evaluated by using the parameters in Eq. (33). The μ-dependent values given at $\mu = 1.45$ Gev ($\mu = 4.4$ Gev) are shown without (with) parentheses.

	fI	fII^2	$fII^3(\mu)$
$m_g = 0.5$ Gev	12.9-17.7i	-7.8	35.0(53.6)
$m_g = 0.2$ Gev	20.7-19.3i	-7.8	35.0(53.6)

TABLE II: Theoretical predictions for the branching ratio of $B \to \chi_{c0}K$ including twist-3 contributions. For comparison, results without twist-3 contributions are listed in the parentheses.

$10^5 \times Br$	$m_g = 0.5$ Gev	$m_g = 0.2$ Gev
$\mu = 1.45$ Gev	30 (5.4)	42 (8.6)
$\mu = 4.40$ Gev	19 (1.7)	24 (2.7)

and qualitatively estimate the size of these corrections. For the spectator corrections, in our approach the dominant contribution comes from soft gluon exchange or quark off-shellness, which are related to the end point singularities when the transverse momentum in the kaon is ignored. It is therefore interesting to examine if the main contributions in Ref. [27] are also from the regions with small virtualities for quarks.

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APPENDIX

In this APPENDIX we will show that the relative off-shellness determined by the gluon propagator in Fig1.(e) or (f) is of the same order as λ in (28). To be explicit, we use the transverse momentum of the spectator quark to regularize the gluon propagator just like that in PQCD approach [28].

Let l denote the momentum of the spectator quark in B meson, then the denominator in the gluon propagator is $(l-k_2)^2 \simeq -(1-z)\xi\bar{y}m_B^2$ if the transverse component $\vec{l}_\perp$ and $\vec{k}_\perp$ are ignored. Then the transverse momenta can be used to regularize the gluon propagator near the end-point region $\bar{y} \to 0$, and the denominator in this non-zero transverse momentum scheme can be written as [28]

$$\begin{aligned} (l-k_2)^2 &\simeq -(1-z)\xi\bar{y}m_B^2 - (\vec{l}_\perp - \vec{k}_\perp)^2 \\ &= -(1-z)m_B^2(\bar{y} + \frac{(\vec{l}_\perp - \vec{k}_\perp)^2}{(1-z)\xi m_B^2}). \end{aligned} \quad (34)$$

Obviously, here the average value $\langle \frac{(\vec{l}_\perp - \vec{k}_\perp)^2}{(1-z)\xi m_B^2} \rangle$ serves as the relative off-shellness λ' for the virtual gluon. Note that all the components of l should be of order $\bar{\Lambda} = m_B - m_b$, it is then easy to determine that $\lambda' \sim O(\bar{\Lambda}/m_B)$, which is of the same order as λ in (28). Furthermore, both λ and λ' are of positive values, hence $f\mathit{\Pi}^{(2,3)}$ in Tab.I do not contain imaginary parts.

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